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HYBRID CONSTANT DIRECTIVITY HORN

INTRODUCTION

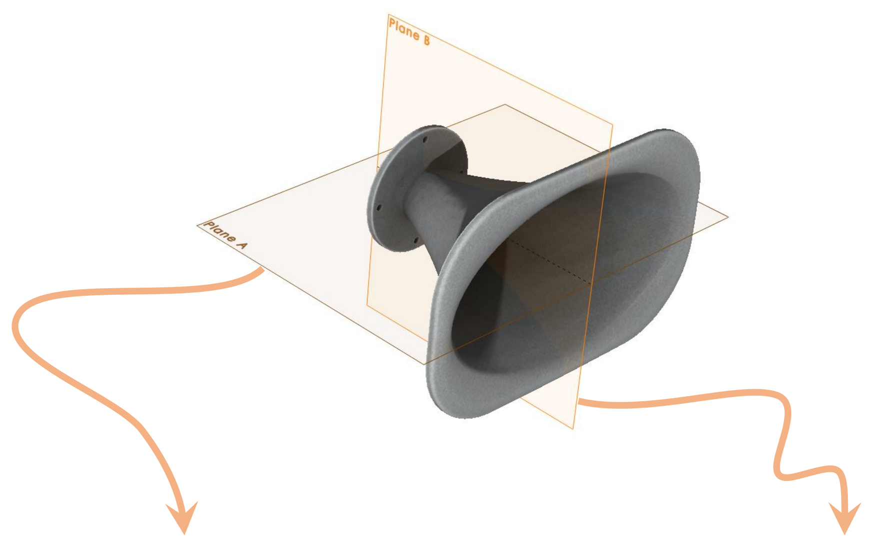
A new horns family is presented, the Hybrid Constant Directivity (HCD), investigating some practical aspects of constant directivity design through physical and FEA 3D prototypes. Horn driver SPL simulations are conducted using a method already presented to the scientific community and here improved, lead to a minimum mismatch between horn simulations and measurements. A detailed directivity and numerical match of the beam-width are examined with a direct SPL comparison among exponential, tractrix and spherical expansions. Then, horns aspect ratio is changed obtaining HCD elliptical and rectangular mouth horns referenced and correlated to the circular one SPL simulation. Also wave-front shapes, mouth diffraction effects and radiation impedances are analyzed. Finally, the mathematical model for calculating HCD horns is disclosed.

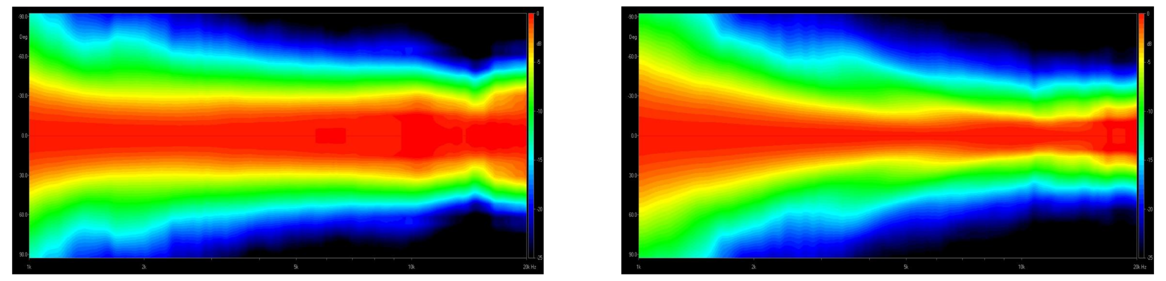
THE *DIGITAL-PHYSICAL* MODEL VALIDATION

Starting from a simulation method presented at the Comsol 2015 Conference, Grenoble (France), which uses the new equation

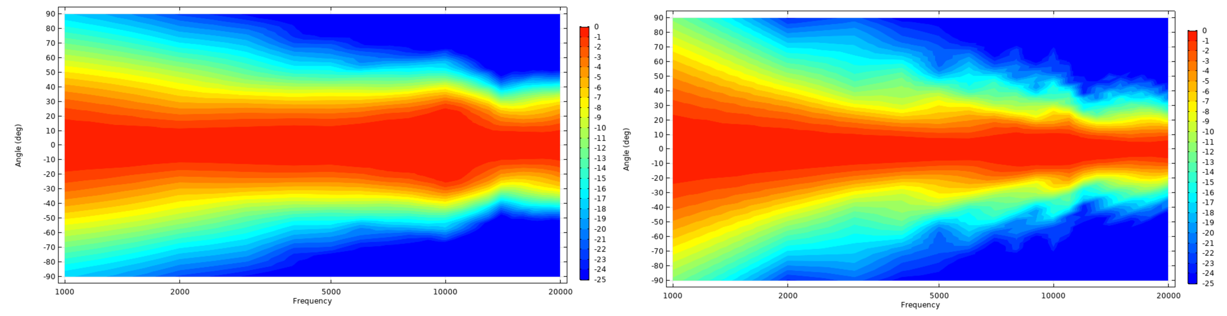
$$L_{pt_{fx}} = 10\log_{10}\left[10 \wedge \left(\frac{L_{pPWT_f}}{20}\right)\cdot p_{sim_{fx}}\right]^2 + \left(L_{pPWT_{fx}} - L_{pPWT_f}\right)$$

for the high frequency *DIGITAL*•**horn**-*PHYSICAL*•**driver** simulation. To predict the physical horn-driver absolute SPL and frequency response, the equation correlates pressures between a compression driver plane wave tube measurement (physical item) and a horn FEA (digital twin). The results are improved here adding the compression driver phase-plug to the horn FEA, then the same model is applied improving horn directivity predictions.

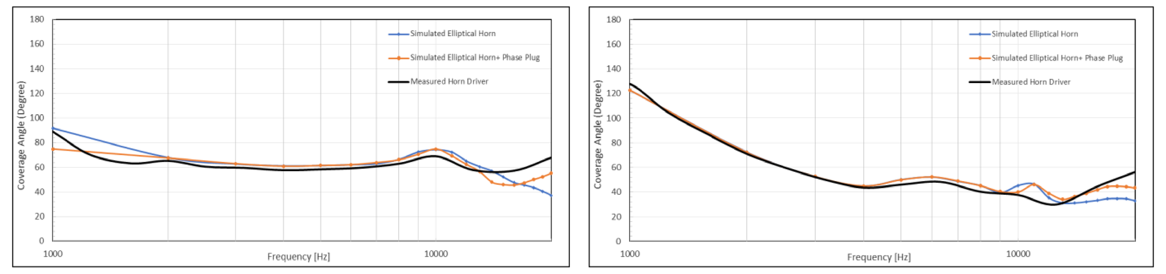




Horn-driver directivity measurements



Horn-driver directivity simulations



Horn-driver beam-width measurement + simulations

HCD MATHEMATICAL MODEL

Starting from the condition of equal volume arrangement

$$Vn_{R=1} - Vn_{R>1} = 0 \qquad (1)$$

where $Vn_{R=1}$ represents the small conical waveguide element with circular shape sides and $Vn_{R>1}$ represents the small conical waveguide element with elliptical shape sides, given by the shape ratio $R > 1$.

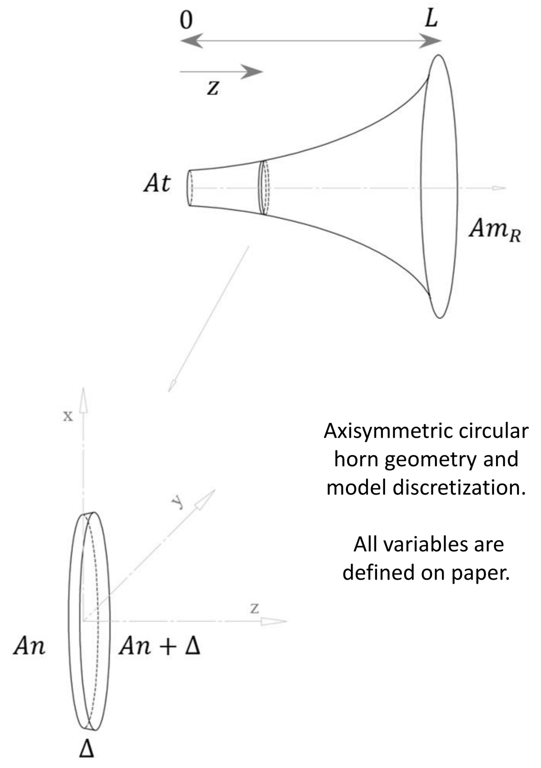
For $\Delta \rightarrow 0$ the condition (1) will be:

$$An_{R=1} - An_{R>1} = 0 \qquad (2)$$

Where $z = kn$ and $0 \leq z \leq L$

$$An = At \xrightarrow{yields} \varepsilon = 0 \qquad (3)$$

$$An = Am_{R>1} \xrightarrow{yields} \varepsilon = \frac{f}{a} \qquad (4)$$

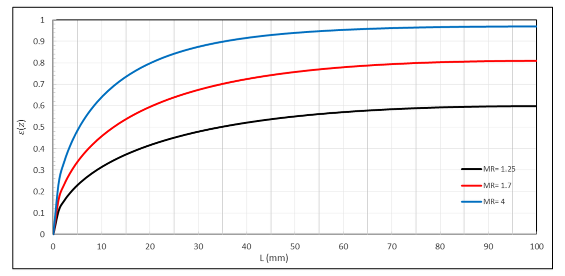


Considering the circular shape as a degeneration of the elliptical shape, characterized by its specific eccentricity ε_n and its linear eccentricity f_n, where f is the distance between the ellipse center and either of its two foci.
Defining a circular mouth horn by the expansion ψn and the rate expansion of ψn for finding the surface area An, it is possible to compute an equivalent elliptical mouth horn defining $\varepsilon_{R>1}(z)$:

$$\varepsilon_{R>1}(z) = \sqrt{1 - \left(\frac{\pi\left(\sqrt{\frac{An}{\pi}} - z\delta\right)^2}{An}\right)^2} \qquad (5)$$

that respect the condition $\varepsilon_{R>1}(0) = 0$
and the condition $\varepsilon_{R>1}(L) = \frac{f}{a}$

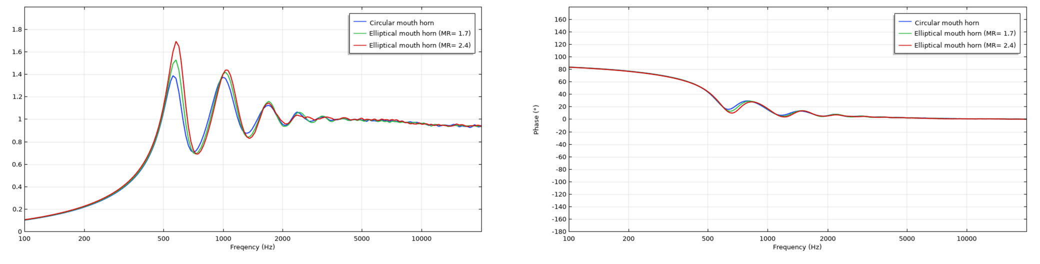
where δ is a term used for the eccentricity progression along the horn length z. Some $\varepsilon_{R>1}(z)$ curves are reported on the right graph for different mouth ratios, in which each curve represents how fast the section transformation is for a given HCD horn mouth ratio.



After some passages we can write the equal volume arrangement in integral form as a function of An and ε. Starting by a known surface area, satisfying the boundary conditions,

$$\int_0^L\int_{-\beta(z)}^{\beta(z)} 2\sqrt{\frac{An}{\pi} - x^2}\,dx\,dz - \int_0^L\int_{-\alpha(z)}^{\alpha(z)} 2\left(\sqrt{\frac{An}{\pi}} - z\delta\right)\sqrt{1 - \frac{x^2(1-\varepsilon^2(z))}{\left(\sqrt{\frac{An}{\pi}} - z\delta\right)^2}}\,dx\,dz = 0$$

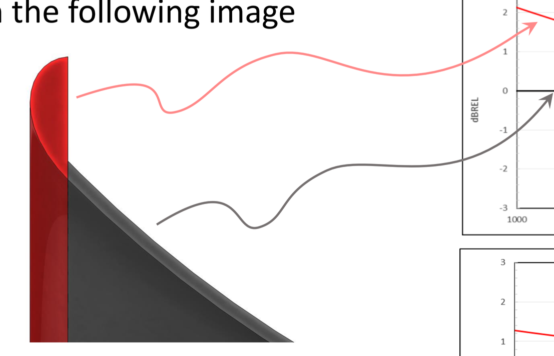
solving them in a system in function of eccentricity, the result is an elliptical HCD horn. An analogous procedure can be used for square and rectangular horns.

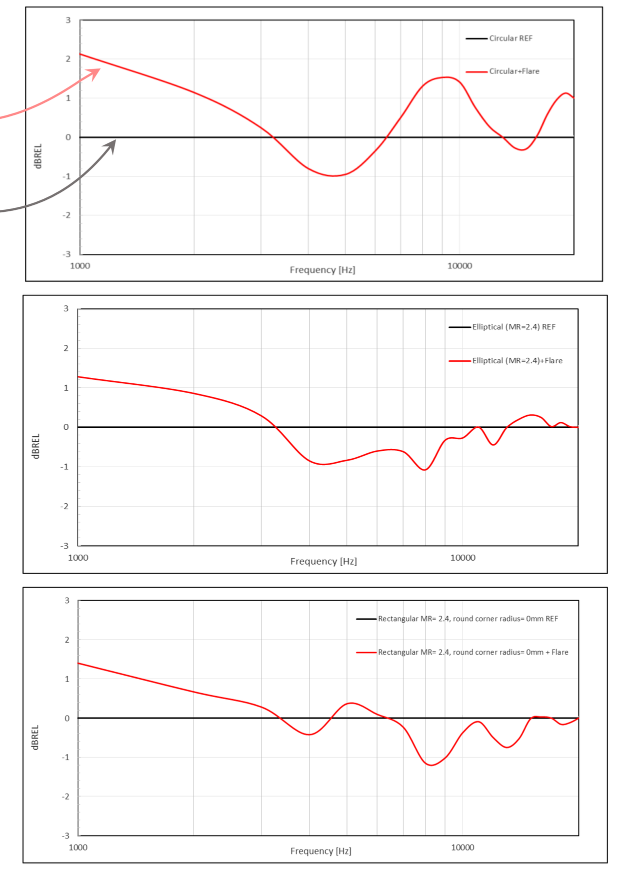


Elliptical mouth HCD horns throat radiation impedances compared to circular mouth

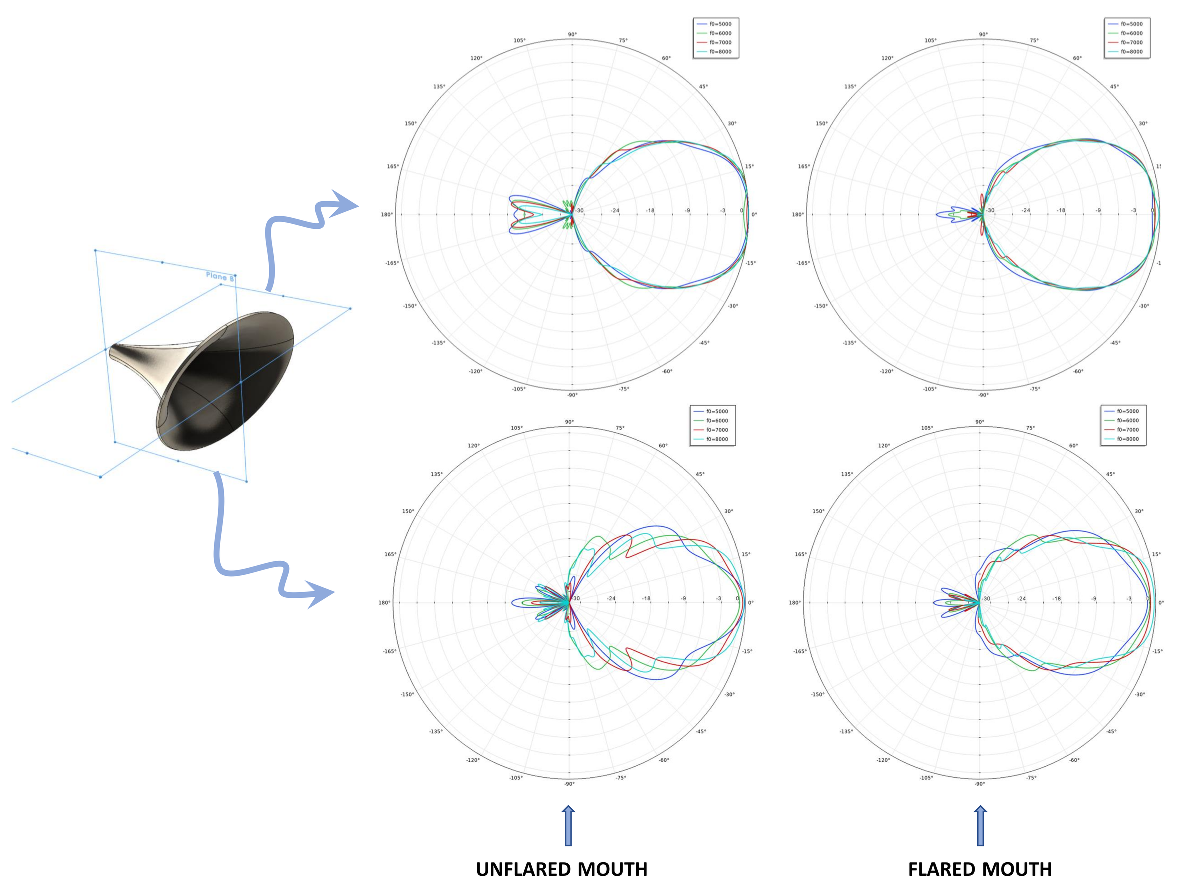
MOUTH DIFFRACTION EFFECTS

Some types of horns have a natural flared mouth (tractrix and spherical expansions for example), while the Hypex family horns have an unflared mouth. Adding the flared end loop to a Hypex horn mouth, red color in the following image





If we change the mouth shape, maintaining the same expansion, the flare has a different behaviour on frequency response curve. For instance a flare is applied on a circular, an elliptical and a rectangular mouth exponential horns, respectively the top, the centre and the bottom right graphs. From the normalized on axis frequency response we can see the deviation adding the flare to the horn (red curves), referenced to the same horn with the unflared mouth (black curves).

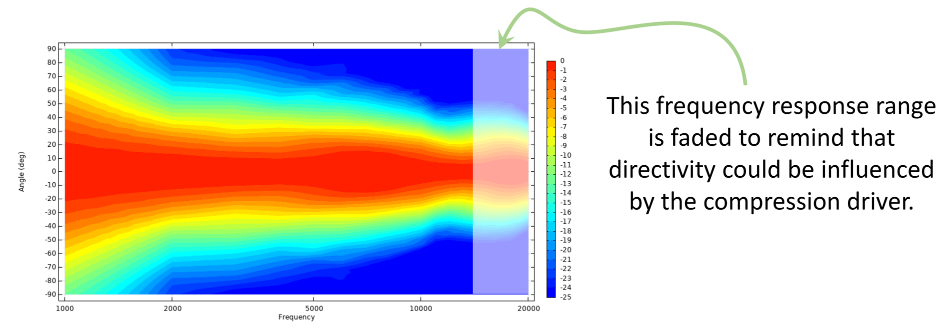


Polar patterns show a different flare impact on the two planes. If Mouth-Ratios> 1, in general for all horns having a different progression on perpendicular side sections, there is not a unique profile to design the flare, but we need to differentiate it along the mouth loop.

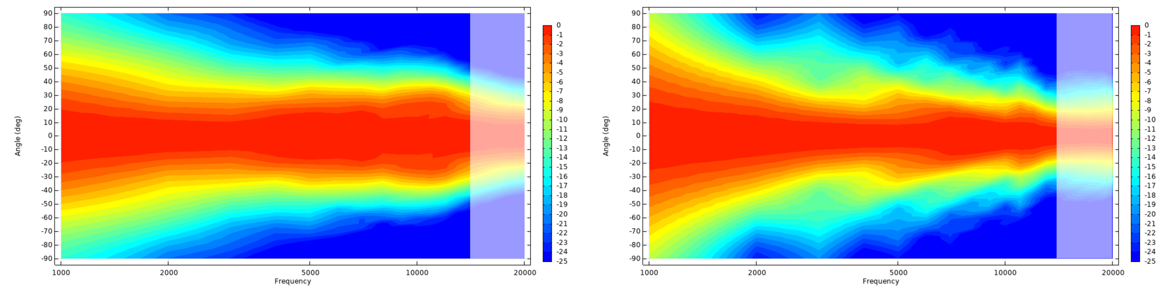
HYBRID CONSTANT DIRECTIVITY (HCD)

HCD transforms a conventional expansion horn (exponential, hyperbolic sine, hyperbolic cosine, catenoidal, tractrix, spherical, etc.) into a constant directivity horn, maintaining the sound characteristic that identify the expansion. HCD can guarantee:

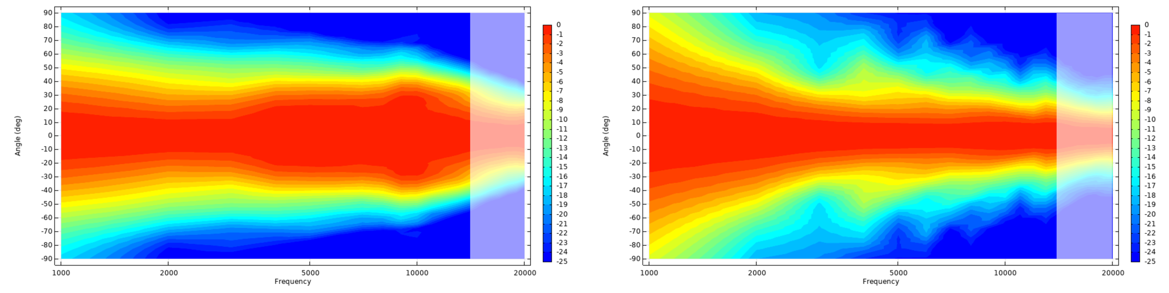
- the horn mathematical expansion we already know;
- a progressive constant directivity on the plane along its mouth major axis;
- an equivalent directivity contour of a circular mouth horn (using the same expansion) on the plane along its mouth minor axis.



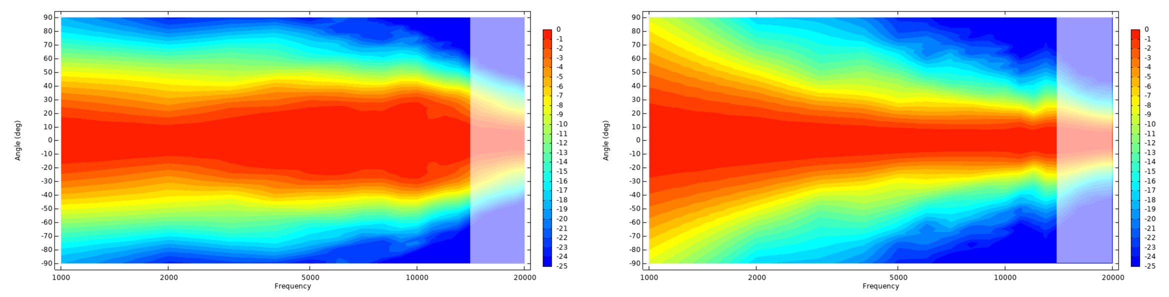
Circular flared mouth exponential horn directivity



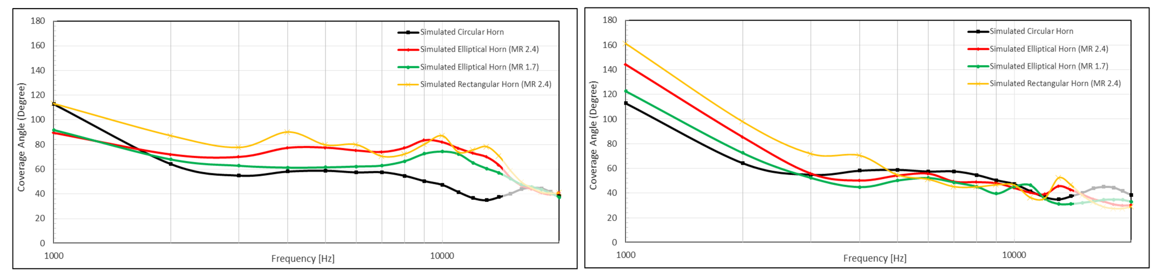
Elliptical flared mouth exponential HCD horn (Mouth Ratio= 1.7) directivity



Elliptical flared mouth exponential HCD horn (Mouth Ratio= 2.4) directivity



Rectangular flared mouth exponential HCD horn (Mouth Ratio= 2.4) directivity



Horns beam-width simulations